April 1, 1886.

Professor STOKES, D.C.L., President, in the Chair.

Dr. John Francis Julius von Haast (elected 1867) was admitted into the Society.

The Presents received were laid on the table, and thanks ordered for them.

The following Papers were read:—

I. "On the Correction to the Equilibrium Theory of Tides for the Continents." I. By G. H. DARWIN, LL.D., F.R.S., Fellow of Trinity College, and Plumian Professor in the University of Cambridge. II. By H. H. TURNER, B.A., Fellow of Trinity College, Cambridge. Received March 12, 1886.

I.

In the equilibrium theory of the tides, as worked out by Newton and Bernouilli, it is assumed that the figure of the ocean is at each instant one of equilibrium.

But Sir William Thomson has pointed out that, when portions of the globe are occupied by land, the law of rise and fall of water given in the usual solution cannot be satisfied by a constant volume of water.*

In Part I of this paper Sir William Thomson's work is placed in a new light, which renders the conclusions more easily intelligible, and Part II contains the numerical calculations necessary to apply the results to the case of the earth.

If m, r, z be the moon's mass, radius vector, and zenith distance; g mean gravity; ρ the earth's mean density; σ the density of water; a the earth's radius; and $\mathfrak{h}$ the height of tide; then, considering only the lunar influence, the solution of the equilibrium theory for an ocean-covered globe is—

$$\frac{\mathfrak{h}}{a}=\frac{3m}{2gr^3}\frac{1}{(1-\frac{3}{5}\sigma/\rho)}(\cos^2 z-\tfrac{1}{3}) \quad \ldots\ldots\ldots\ldots (1)$$

* Thomson and Tait's "Nat. Phil.," 1883, § 808.

This equilibrium law would still hold good when the ocean is interrupted by continents, if water were appropriately supplied to or exhausted from the sea as the earth rotates.

Since when water is supplied or exhausted the height of water will rise or fall everywhere to the same extent, it follows that the rise and fall of tide, according to the revised equilibrium theory, must be given by—

$$\frac{\mathfrak{h}}{a}=\frac{3ma}{2gr^3}\frac{1}{1-\frac{3}{5}\sigma/\rho}(\cos^2 z-\tfrac{1}{3})-\alpha \quad \ldots\ldots\ldots\ldots (2)$$

where α is a constant all over the earth for each position of the moon relatively to the earth, but varies for different positions.

Let Q be the fraction of the earth's surface which is occupied by sea; let λ be the latitude and l the longitude of any point; and let ds stand for $\cos\lambda d\lambda dl$, an element of solid angle. Then we have—

$$4\pi Q=\iint ds$$

integrated all over the oceanic area.

The quantity of water which must be subtracted from the sea, so as to depress the sea level everywhere by $a\alpha$, is $4\pi a^3\alpha Q$; and the quantity required to raise it by the variable height $\frac{3ma^2}{2gr^3}\frac{\cos^2 z-\frac{1}{3}}{1-\frac{3}{5}\sigma/\rho}$ is the integral of this function, taken all over the ocean. But since the volume of water must be constant, continuity demands that—

$$\alpha=\frac{3ma}{2g(1-\frac{3}{5}\sigma/\rho)r^3}\cdot\frac{1}{4\pi Q}\iint(\cos^2 z-\tfrac{1}{3})ds \quad \ldots\ldots\ldots (3)$$

integrated all over the ocean.

On substituting this value of α in (2) we shall obtain the law of rise and fall.

Now if λ, l be the latitude and W. longitude of the place of observation; h the Greenwich westward hour-angle of the moon at the time and place of observation; and δ the moon's declination, it is well known that—

$$\cos^2 z-\tfrac{1}{3}=\tfrac{1}{2}\cos^2\lambda\cos^2\delta\cos 2(h-l)+\sin 2\lambda\sin\delta\cos\delta\cos(h-l)$$
$$+\tfrac{3}{2}(\tfrac{1}{3}-\sin^2\delta)(\tfrac{1}{3}-\sin^2\lambda) \quad \ldots\ldots\ldots\ldots (4)$$

We have next to introduce (4) under the double integral sign of (3), and integrate over the ocean.

To express the result conveniently, let—

$$\frac{1}{4\pi Q}\iint \cos^2\lambda \cos 2l\, ds = \cos^2\lambda_2 \cos 2l_2,\ \frac{1}{4\pi Q}\iint \cos^2\lambda \sin 2l\, ds = \cos^2\lambda_2 \sin 2l_2,$$

$$\frac{1}{4\pi Q}\iint \sin 2\lambda \cos l\, ds = \sin 2\lambda_1 \cos l_1,\ \frac{1}{4\pi Q}\iint \sin 2\lambda \sin l\, ds = \sin 2\lambda_1 \sin l_1,$$

$$\frac{1}{4\pi Q}\iint (\tfrac{3}{2}\sin^2\lambda - \tfrac{1}{2})ds = \tfrac{3}{2}\sin^2\lambda_0 - \tfrac{1}{2} \quad \ldots\ldots\ldots \quad (5)$$

the integrals being taken over the oceanic area.

These five integrals are called by Sir William Thomson 𝔄, 𝔅, ℭ, 𝔇, 𝔈, but by introducing the five auxiliary latitudes and longitudes, λ_2, l_2, λ_1, l_1, λ_0 we shall find for the conclusions an easily intelligible physical interpretation.

It may be well to observe that (5) necessarily give real values to the auxiliaries. For consider the first integral as a sample :—

Every element of $\iint\cos^2\lambda \cos 2l\, ds$ is, whether positive or negative, necessarily numerically less than the corresponding element of $4\pi Q$, and therefore, even if all the elements of the former integral were taken with the same sign, $(4\pi Q)^{-1}\iint\cos^2\lambda \cos 2l\, ds$ would be numerically less than unity, and *à fortiori* in the actual case it is numerically less than unity.

Now using (5) in obtaining the value of $\iint(\cos^2 z - \frac{1}{3})ds$, and substituting in (3), we have—

$$\frac{\mathfrak{h}}{a} \div \frac{3ma}{2g(1-\frac{3}{5}\sigma/\rho)r^3} = \tfrac{1}{2}\cos^2\delta[\cos^2\lambda \cos 2(h-l) - \cos^2\lambda_2 \cos 2(h-l_2)]$$

$$+ \sin 2\delta[\sin\lambda \cos\lambda \cos(h-l) - \sin\lambda_1 \cos\lambda_1 \cos(h-l_1)]$$

$$+ \tfrac{3}{2}(\tfrac{1}{3} - \sin^2\delta)(\sin^2\lambda_0 - \sin^2\lambda) \quad \ldots\ldots\ldots\ldots \quad (6)$$

The first term of (6) gives the semi-diurnal tide, the second the diurnal, and the third the tide of long period.

The meaning of the result is clear. The latitude and longitude λ_2, l_2 is a certain definite spot on the earth's surface which has reference to the semi-diurnal tide. Similarly λ_1, l_1 is another definite spot which has reference to the diurnal tide; and λ_0 is a definite parallel of latitude which has reference to the tide of long period.

From inspection we see that at the point λ_2, l_2 the semi-diurnal tide is evanescent, and that at the point λ_2, $l_2+90°$ there is doubled tide, as compared with the uncorrected equilibrium theory. At the place λ_1, l_1 the diurnal tide is evanescent, and at $-\lambda_1$, l_1 there is doubled diurnal tide.

In the latitude λ_0 the long period tide is evanescent, and in latitude (sometimes imaginary) arc $\sin\sqrt{\{\frac{2}{3} - \sin^2\lambda_0\}}$ there is doubled long period tide.

Many or all of these points may fall on continents, so that the evanescence or doubling may only apply to the algebraical expressions, which are, unlike the sea, continuous over the whole globe. But now let us consider more precisely what the points are.

It is obvious that the latitude and longitude λ_2 and l_2, being derived from expressions for $\cos^2\lambda_2 \cos 2l_2$ and $\cos^2\lambda_2 \sin 2l_2$, really correspond with four points whose latitudes and longitudes are—

$$\lambda_2, l_2;\ -\lambda_2, l_2;\ \lambda_2, l_2+180°;\ -\lambda_2, l_2+180°.$$

Thus there are four points of evanescent semi-diurnal tide, situated on a single great circle or meridian, in equal latitudes N. and S., and antipodal two and two. Corresponding to these four, there are four points of doubled semi-diurnal tide, whose latitudes and longitudes are—

$$\lambda_2, l_2+90°;\ -\lambda_2, l_2+90°;\ \lambda_2, l_2+270°;\ \lambda_2, l_2+270°,$$

and these also are on a single great circle or meridian, at right angles to the former great circle, and are in the same latitudes N. and S. as are the places of evanescence, and are antipodal two and two.

Passing now to the case of the diurnal tide we see that λ_1, l_1, being derived from expressions for $\sin 2\lambda_1 \cos l_1$ and $\sin 2\lambda_1 \sin l_1$, really correspond with four points whose latitudes and longitudes are—

$$\lambda_1, l_1;\ -\lambda_1, l_1+180°;\ 90°-\lambda_1, l_1;\ -90°+\lambda_1, l_1+180°.$$

Thus there are four points of evanescent diurnal tide, situated on a single great circle or meridian, two of them are in one quadrant in complemental latitudes, and antipodal to them are the two others. Corresponding to these four there are four points of doubled diurnal tide lying in the same great circle or meridian, and situated similarly with regard to the S. pole as are the points of evanescence with regard to the N. pole; their latitudes and longitudes are—

$$-\lambda_1, l_1;\ \lambda_1, l_1+180°;\ -90°+\lambda_1, l_1;\ 90°-\lambda_1, l_1+180°.$$

Lastly, in the case of the long period tide, it is obvious that the latitude λ_0 is either N. or S., and that there are two parallels of latitude of evanescent tide. In case $\sin^2\lambda_0$ is less than $\frac{2}{3}$, or λ_0 less than 54° 44′, there are two parallels of latitude of doubled tide of long period in latitude $\frac{2}{3}$ arc sin $\sqrt{\{\frac{2}{3}-\sin^2\lambda_0\}}$.

From a consideration of the integrals, it appears that as the continents diminish towards vanishing, the four points of evanescent and the four points of doubled semi-diurnal tide close in to the pole, two of each going to the N. pole, and two going to the S. pole; also one of the points of evanescent and one of doubled diurnal tide go to the N. pole, a second pair of points of evanescence and of doubling go to the S. pole, a third pair of points of evanescence and of doubling coalesce on the equator, and a fourth pair coalesce at the antipodes of the third pair; lastly, in the case of the tides of long period the circles of evanescent tide tend to coalesce with the circles of doubled tide, in latitudes 35° 16′ N. and S.

We are now in a position to state the results of Thomson's corrected theory by comparison with Bernouilli's theory.

Consider the semi-diurnal tide on an ocean-covered globe, then at the four points on a single meridian great circle which correspond to the points of evanescence on the partially covered globe, the tide has the same height; and at any point on the partially covered globe the semi-diurnal tide is the excess (interpreted algebraically) of the tide at the corresponding point on the ocean-covered globe above that at the four points.

A similar statement holds good for the diurnal and tides of long period.

By laborious quadratures Mr. Turner has evaluated in Part II the five definite integrals on which the corrections to the equilibrium theory, as applied to the earth, depend.

The values found show that the points of evanescent semi-diurnal tide are only distant about 9° from the N. and S. poles; and that of the four points of evanescent diurnal tide two are close to the equator, one close to the N. pole, and the other close to the S. pole; lastly, that the latitudes of evanescent tide of long period are 34° N. and S., and are thus but little affected by the land.

Thus in all cases the points of evanescence are situated near the places where the tides vanish when there is no land. It follows, therefore, that the correction to the equilibrium theory for land is of no importance.

G. H. D.

II.

For the evaluation of the five definite integrals, called by Sir William Thomson 𝔄, 𝔅, ℭ, 𝔇, 𝔈, and represented in the present paper by functions of the latitudes and longitudes λ_0, λ_1, λ_2, and l_1, l_2, respectively similar in form to the functions of the "running" latitude and longitude to be integrated, it is necessary to assume some redistribution of the land on the earth's surface, differing as little as possible from the real distribution, and yet with a coast line amenable to mathematical treatment. The integrals are to be taken over the whole ocean, but since the value of any of them taken over the whole sphere is zero, the part of any due to the sea is equal to the part due to the land with its sign changed; and since there is less land than sea, it will be more convenient to integrate over the land, and then change the sign.

Unless specially mentioned, we shall hereafter assume that the integration is taken over the land.

The last of the integrals has already been evaluated by Professor Darwin,* with an approximate coast line, which follows parallels of latitude and longitude alternately.

His distribution of land is given in the following table :—

N. lat.	W. long.	E. long.
Lat. 80° to 90°	20° to 50°.	
70 „ 80	22° to 55° : 85° to 115°.	55° to 60° : 90° to 110°.
60 „ 70	35° to 52° : 65° to 80° : 90° to 165°.	10° to 180°.
50 „ 60	0° to 6° : 60° to 78° : 90° to 130°.	10° to 140° : 155° to 160°.
40 „ 50	0° to 5° : 65° to 123°.	0° to 135°.
30 „ 40	0° to 8° : 78° to 120°.	0° to 120° : 135° to 138°.
20 „ 30	0° to 15° : 80° to 82° : 97° to 110°.	0° to 118°.
10 „ 20	0° to 17° : 87° to 95°.	0° to 50° : 75° to 85° : 95° to 108° : 122° to 125°.
0 „ 10	53° to 78°.	0° to 48° : 98° to 105° : 112° to 117°.
S. lat.	**W. long.**	**E. long.**
0° to 10°	37° to 80°	12° to 40° : 110° to 130°.
10 „ 20	37 „ 74	12° to 38° : 45° to 50° : 126° to 144°.
20 „ 30	45 „ 71	15° to 33° : 115° to 151°.
30 „ 40	55 „ 73	20° to 23° : 132° to 140°.
40 „ 50	65 „ 73	170° to 172°.
50 „ 60	67 „ 72	——
60 „ 70	55 „ 65	120° to 130°.
70 „ 80	about 20° of longitude.	
80 „ 90	„ 180° „	

N.B.—*The Mediterranean, being approximately a lake, is treated as land.*

The limits of the 20° and 180° of longitude between S. latitudes 70° and 90° are not specified. For the evaluation of the last integral this is not necessary, for restricting

$$\iint (3\sin^2\lambda - 1)\cos\lambda \, d\lambda \, dl$$

to a representative portion of the land bounded by parallels λ_1 and λ_2, l_1 and l_2, we get $-\frac{1}{4}(l_2-l_1)\Big[\sin\lambda + \sin 3\lambda\Big]_{\lambda_1}^{\lambda_2} \times \frac{\pi}{180}$; and similarly for Q; so that if t_1 and t_2 be the number of degrees of longitude N. and S.

* Thomson and Tait's "Nat. Phil.," 1883, § 808.

of the equator respectively between latitudes λ_1 and λ_2, the last of the integrals becomes

$$\frac{\Sigma\frac{1}{4}(t_1+t_2)\Big[\sin\lambda+\sin 3\lambda\Big]_{\lambda_1}^{\lambda_2}}{720-\Sigma(t_1+t_2)\Big[\sin\lambda\Big]_{\lambda_1}^{\lambda_2}}.$$

But for (*e.g.*)

$$\int_{\lambda_1}^{\lambda_2}\int_{l_1}^{l_2}\cos^2\lambda\cos 2l\cos\lambda\, d\lambda\, dl = \frac{1}{24}\Big[9\sin\lambda+\sin 3\lambda\Big]_{\lambda_1}^{\lambda_2}\Big[\sin 2l\Big]_{l_1}^{l_2}$$

the actual limits l_2 and l_1 must be given, and not merely their difference.

It is, however, obvious, on inspection of these integrals, that the land in high latitudes affects them but little; and we shall not lose much by neglecting entirely the Antarctic continent in their evaluation.

This evaluation is reduced by the above process to a series of multiplications, and on performing them the following values of $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$, and Q are obtained on the two hypotheses.

(1.) That there is as much Antarctic land as is given in the schedule, which is, however, only taken into account in the last integral $\mathfrak{E}$, and the common denominator $4\pi Q$ of each.

(2.) That there is no land between S. latitude 80° and the pole.

The value of Q is given in terms of the whole surface, and represents the fraction of that surface occupied by land; it must be remembered that the Mediterranean Sea is treated as land. Professor Darwin quotes Rigaud's estimate* as 0·266:—

	1st hypothesis.	2nd hypothesis.
$\mathfrak{A}$..........	+0·03023	+0·03008
$\mathfrak{B}$..........	+0·00539	+0·00537
$\mathfrak{C}$..........	−0·01975	−0·01965
$\mathfrak{D}$..........	+0·02910	+0·02895
$\mathfrak{E}$..........	−0·01520	−0·00486
Q..........	0·283	0·278

These results for $\mathfrak{E}$ and Q have already been given by Professor Darwin in "Thomson and Tait's Natural Philosophy," and I have found them correct.

* "Trans. Cam. Phil. Soc.," vol. vi.

We then find for the set of latitudes and longitudes of evanescent tide :—

Nature of tide.		1st hypothesis.	2nd hypothesis.
Long period	lat. λ_0	34° 39′ N.	35° 4′ N.
Diurnal .... {	lat. λ_1 long. l_1	1° 0′ S. 55 50 E.	1° 0′ S. 55 50 E.
Semi-diurnal .... {	lat. λ_2 long. l_2	79° 54′ N. 5 3 W.	79° 56′ N. 5 4 W.

The other points of evanescence are of course easily derivable from these, as shown in the first part of this paper.

As a slightly closer approximation to truth, I have calculated these integrals on another supposition. There are cases where lines satisfying the equations

$$l=\text{const. or } \lambda=\text{const.}$$

diverge somewhat widely from the actual coast line, but a line

$$\pm al\pm b\lambda=\text{const.}$$

(where a and b are small integers) can be found following it more faithfully. An approximate coast line of the land on the earth is defined in the following schedule, west longitudes and north latitudes being considered positive.

Limits of longitude (l).		Equation.		Limits of latitude (λ).
+ 20° to + 10°	....	$-\lambda=l-40$	....	+20° to +30°
——	....	$l=10$	....	+30 „ +40
+ 10 „ − 23	....	$-\lambda=l-50$	..	+40 „ +73
− 23 „ +120	....	$\lambda=73$	....	——
——	....	$l=120$	....	+73 „ +80
+120 „ + 20	....	$\lambda=80$	....	——
——	....	$l=20$	....	+80 „ +70
+ 20 „ + 50	....	$-3\lambda=l-230$	....	+70 „ +60
+ 50 „ + 70	....	$\lambda=l+10$	....	+60 „ +80
+ 70 „ + 80	....	$\lambda=80$	....	——
+ 80 „ + 50	....	$\lambda=l$	....	+80 „ +50
+ 50 „ + 90	....	$-2\lambda=l-150$	....	+50 „ +30
+ 90 „ +100	....	$\lambda=30$	....	——
+100 „ + 80	....	$\lambda=l-70$	....	+30 „ +10
+ 80 „ + 70	....	$\lambda=10$	....	——
+ 70 „ + 30	....	$2\lambda=l-50$	....	+10 „ −10
+ 30 „ + 73	....	$-\lambda=l-20$	....	−10 „ −53
——	....	$l=73$	....	−53 „ −14

Limits of longitude (l).		Equation.		Limits of latitude (λ).
$+78°$ to $+80°$	....	$\lambda=2l-160$	....	$-14°$ to $0°$
$+80$ „ $+140$	....	$\lambda=l-80$	....	0 „ $+60$
$+140$ „ -150	....	$\lambda=60$	....	——
-150 „ -100	....	$-\lambda=l+90$	....	$+60$ „ $+10$
-100 „ -90	....	$+\lambda=l+110$	....	$+10$ „ $+20$
-90 „ -80	....	$-\lambda=l+70$	....	$+20$ „ $+10$
-80 „ -65	....	$\lambda=l+90$	....	$+10$ „ $+25$
-65 „ -40	....	$-\lambda=l+40$	....	$+25$ „ 0
——	..	$l=-40$	....	0 „ -20
-40 „ -20	....	$-\lambda=l+60$	....	-20 „ -40
-20 „ $-8\frac{3}{4}$	....	$\lambda=4l+40$	....	-40 „ $+5$
$-8\frac{3}{4}$ „ $+12\frac{1}{2}$	....	$\lambda=5$	....	——
$+12\frac{1}{2}$ „ $+20$	....	$\lambda=2l-20$	....	$+5$ „ $+20$

New Guinea.

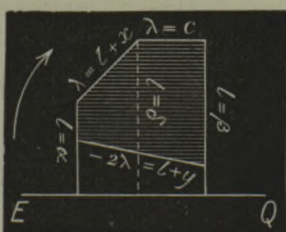
-130 to -150	....	$2\lambda=l+130$	....	0 to -10
-150 „ -140	.. .	$\lambda=-10$	....	——
-140 „ -130	....	$\lambda=l+130$	....	-10 „ 0

Australia.

-140 to -150	....	$\lambda=l+130$	....	-10 to -20
——	....	$l=-150$	....	-20 „ -35
-150 „ -115	....	$\lambda=-35$	....	——
——	....	$l=-115$	....	-35 „ $-22\frac{1}{2}$
-115 „ -140	....	$-2\lambda=l+160$	....	$-22\frac{1}{2}$ „ -10

It will be seen that it is only rarely necessary to depart from the forms of equation $\pm\lambda=l+x$ and the two original forms $\lambda=$const. $l=$ const. to represent the coast line with considerable accuracy. There are still left one or two outlying portions, of which mention will be made later.

Now supposing we are to find the value of the first integral for the portion of land indicated by the shaded portion of the diagram, E, Q being the equator:



the equations to its boundaries being written at the side of each.

We have

$$\iint \cos^3\lambda\, d\lambda \cos 2l\, dl = \frac{1}{12}\int \Big[9\sin\lambda + \sin 3\lambda\Big]_{\lambda_1}^{\lambda_2} \cos 2l\, dl$$

$$= \frac{1}{12}\int_{\alpha}^{\delta} \{9\sin(l+x) + \sin 3(l+x)\}\cos 2l\, dl$$

$$+ \int_{\delta}^{\beta} (9\sin c + \sin 3c)\cos 2l\, dl$$

$$+ \int_{\beta}^{\alpha} -\{9\sin\tfrac{1}{2}(l+y) + \sin\tfrac{3}{2}(l+y)\}\cos 2l\, dl.$$

We may thus simply travel round the boundary omitting the places where λ=constant: being careful to go round all the pieces of land in the same direction. If we suppose $l=\alpha$ to be the meridian of Greenwich, and the land to be in the northern hemisphere, the direction indicated above is the wrong one for obtaining the value of the integrals over the land, for the longitudes increase to the left; but by following this direction we shall obtain the values over the sea as is in reality required.

The result of integration has, of course, a different form for each form of the relation between l and λ representing the boundary. In computing the numerical values of the integrals, it is convenient to consider together all the parts of the boundary represented by similar equations.

Below are given as representative the forms which the numerator of the first integral $\mathfrak{J}$ assumes for different forms of the boundary, the quantities within square brackets being taken within limits.

Form.		Value of Integral.
$\mp\lambda = l+x$	....	$\pm\frac{1}{24}[\frac{1}{5}\cos(5l+3x) + 3\cos(3l+x) + \cos(l+3x)$ $-9\cos(-l+x)]$
$\lambda = x$	....	$+\frac{1}{24}(9\sin x + \sin 3x)[\sin 2l]$
$l = x$	....	Zero
$\lambda = 2l+x$	....	$-\frac{1}{24}[\frac{9}{4}\cos(4l+x) - 9l\sin x + \frac{1}{8}\cos(8l+3x)$ $+\frac{1}{4}\cos(4l+3x)]$
$\lambda = 4l+x$	....	$-\frac{1}{24}[\frac{3}{2}\cos(6l+x) + \frac{9}{2}\cos(2l+x) + \frac{1}{14}\cos(14l+3x)$ $+\frac{1}{10}\cos(10l+3x)]$
$\mp 2\lambda = l+x$	....	$\pm\frac{1}{24}[\frac{18}{5}\cos\frac{1}{2}(5l+x) - 6\cos\frac{1}{2}(-3l+x)$ $+\frac{2}{7}\cos\frac{1}{2}(7l+3x) - 2\sin\frac{1}{2}(-l+3x)]$
$-3\lambda = l+x$	....	$+\frac{1}{24}[\frac{27}{7}\cos\frac{1}{3}(7l+x) - \frac{27}{5}\cos\frac{1}{3}(-5l+x)$ $+\frac{1}{3}\cos(3l+x) - \cos(-l+x)]$

Evaluating these integrals on this supposition, we obtain

	1st hypothesis.	2nd hypothesis.
𝔄..........	+0·02119	+0·02110
𝔅..........	+0·00778	+0·00775
ℭ..........	−0·01890	−0·01882
𝔇..........	+0·03159	+0·03128
𝔈..........	−0·04364	−0·03319
Q..........	0·283	0·278

It will be noticed that the values of Q are exactly the same as before.

From these we deduce

Nature of tide.		1st hypothesis.	2nd hypothesis.
Long period	lat. λ_0	33° 29′ N.	33° 55′ N.
Diurnal .... {	lat. λ_1 long. l_1	1° 3′ S. 59 7 E.	1° 3′ S. 58 58 E.
Semi-diurnal .... {	lat. λ_2 long. l_2	81° 22′ N. 10 5 W.	81° 23′ N. 10 5 W.

The agreement of these values of the quantities with the values calculated on the previous supposition is not quite so close as I anticipated, but it should be remarked that the numerators of the quantities 𝔄, 𝔅, ℭ, 𝔇, 𝔈 are the differences of positive and negative quantities of very much greater magnitude, as becomes obvious on proceeding to the numerical calculation; and thus a comparatively small change in one of the large compensating quantities, due to large tracts of land in different portions of the globe, affects the integrals a considerable extent.

In this connexion I was led to investigate the effect of counting various small islands and promontories as sea, or small bays and straits as land. For instance, a portion of sea in the neighbourhood of Behring's Straits is included as land, and a corresponding correction must be applied to the integrals. This correction I have *estimated* as follows:—The area of the sea is estimated in square degrees, by drawing lines on a large map corresponding to each degree of latitude and longitude and counting the squares covered by sea, fractions of a square to one decimal place being included, though the tenths have been neglected in the concluded sum. This area has then been multiplied by the value of (say) $\cos^3\lambda \cos 2l$

for the approximate centre of gravity of the portion, to find an approximate value of the integral $\iint \cos^3\lambda \cos 2l \cos \lambda dl$ over its surface.

By drawing the assumed coast line on a map, it will become obvious that such corrections may be applied for the following portions, defined by the latitude and longitude of their centres of gravity; remarking that when there is a portion of land which may be fairly considered to compensate a portion of sea in the immediate neighbourhood, no correction has been applied. For instance, it would be seen that part of the Kamschatkan Peninsula is excluded from the coast line, and part of the Sea of Okhotsk is included; but these will produce nearly equal effects on the integrals in opposite directions, and are thus left out of consideration.

Area in square degrees.	Longitude.	Latitude.
+160	+172°	+64°
+240	+150	$+71\frac{1}{2}$
+166	+ 85	+60
+ 80	+ 60	+52
+ 68	+ 85	+ 9
− 20	+ 75	+21
− 69	+ 55	+ 4
+ 43	+ 34	−11
+ 22	− 37	−20
− 48	− 47	−19
+ 65	− 53	+18
− 16	−107	+13
− 34	−102	+ 2
− 49	−114	+ 1
− 27	−123	+12
− 11	−118	− 5
− 43	−138	+36
− 39	−173	−42

N.B.—*Land-areas are considered positive, sea negative.*

We then find the following corrected values of the integrals:—

	1st hypothesis.	2nd hypothesis.
$\mathfrak{A}$	+0·02237	+0·02247
$\mathfrak{B}$	+0·00230	+0·00231
$\mathfrak{C}$	−0·01952	−0·01961
$\mathfrak{D}$	+0·02665	+0·02676
$\mathfrak{E}$	−0·01775	−0·02810
Q	0·279	0·274

and finally the following values of the latitudes and longitudes of evanescent tides:—

Nature of tide.		1st hypothesis.	2nd hypothesis.
Long period	lat. λ_0	34° 33′ N.	34° 7′ N.
Diurnal .... {	lat. λ_1 long. l_1	0° 57′ S. 53 47 E.	0° 57′ S. 53 46 E.
Semi-diurnal .... {	lat. λ_2 long. l_2	81° 23′ N. 2 56 W.	81° 21′ N. 2 56 W.

The estimation of corrections due to these supplementary portions has been checked in two cases by a detailed extension of the method of square blocks of land used previously for evaluation of the whole integrals; that is to say, two of these portions were separately divided into square degrees (instead of squares whose sides were each ten degrees), and the integral evaluated in a similar manner to that previously described. The agreement of the values so calculated with those obtained by the above method of estimation was sufficiently exact to justify a certain confidence in the close agreement of the finally corrected values of the integrals with their theoretically perfect values.

H. H. T.

II. "Description of Fossil Remains of two Species of a Megalanian Genus (*Meiolania*, Ow.), from Lord Howe's Island." By Sir Richard Owen, K.C.B., F.R.S. Received March 15, 1886.

(Abstract.)

In a scientific survey by the Department of Mines, New South Wales, of Lord Howe's Island, fossil remains were obtained which were transmitted to the British Museum of Natural History, and were confided to the author for determination and description.

These fossils, referable to the extinct family of horned Saurians described in former volumes of the "Philosophical Transactions"* under the generic name *Megalania*, form the subject of the present paper. They represent species smaller in size than *Megalania prisca*, Ow., and with other differential characters on which an allied genus *Meiolania* is founded. Characters of an almost entire skull with part of the lower jaw-bone, of some vertebræ and parts of the scapula and pelvic arches, are assigned to the species *Meiolania*

* Vol. 149, 1858, p. 43; *ib.*, 1880, p. 1037; *ib.*, 1881, p. 1037.