Thalén and that given by Boisbaudran for the bright line to which we assume it to correspond, it was determined in the way above described, and must not be taken to have more importance. By our method of working, the very high dispersive power required for the specific identification of any substance by the determination of an individual wave-length is avoided; as we depend on the greatly diminished likelihood of error when several groups of lines of the same substance are seen to be continually present at the time of one or more reversals. It was the necessity of being able to rapidly sweep the entire spectrum for the above purpose that caused us to limit the dispersion. A large majority of the wave-lengths given by Thalén were obtained by means of the moderate dispersion of one bisulphide of carbon prism, a less dispersion than we have used; and it would be incorrect to suppose that no enduring work in this field of spectroscopy can be effected except with the enormous dispersive power which Mr. Lockyer recommends.

VIII. "The Determination of the Secular Effects of Tidal Friction by a Graphical Method." By G. H. DARWIN, M.A., formerly Fellow of Trinity College, Cambridge. Communicated by J. W. L. GLAISHER, M.A., F.R.S. Received May 31, 1879.

Suppose an attractive particle or satellite of mass m to be moving in a circular orbit, with an angular velocity Ω, round a planet of mass M, and suppose the planet to be rotating about an axis perpendicular to the plane of the orbit, with an angular velocity n; suppose, also, the mass of the planet to be partially or wholly imperfectly elastic or viscous, or that there are oceans on the surface of the planet; then the attraction of the satellite must produce a relative motion in the parts of the planet, and that motion must be subject to friction, or, in other words, there must be frictional tides of some sort or other. The system must accordingly be losing energy by friction, and its configuration must change in such a way that its whole energy diminishes.

Such a system does not differ much from those of actual planets and satellites, and, therefore, the results deduced in this hypothetical case must agree pretty closely with the actual course of evolution, provided that time enough has been and will be given for such changes.

Let C be the moment of inertia of the planet about its axis of rotation;

r the distance of the satellite from the centre of the planet;

h the resultant moment of momentum of the whole system;

e the whole energy, both kinetic and potential of the system.

It will be supposed that the figure of the planet and the distribution of its internal density are such that the attraction of the satellite causes no couple about any axis perpendicular to that of rotation.

Then the two bodies revolve in circles about their common centre of inertia with an angular velocity Ω, and, therefore, the m. of m. of orbital motion is

$$M\left(\frac{mr}{M+m}\right)^2\Omega+m\left(\frac{Mr}{M+m}\right)^2\Omega=\frac{Mm}{M+m}r^2\Omega.$$

Let μ be attraction between unit masses at unit distance.

Then, by the law of periodic times, in a circular orbit, $\Omega^2r^3=\mu(M+m)$ whence

$$\Omega r^2=\mu^{\frac{2}{3}}(M+m)\Omega^{-\frac{1}{3}}.$$

And the m. of m. of orbital motion$=\mu^{\frac{2}{3}}Mm(M+m)^{-\frac{1}{3}}\Omega^{-\frac{1}{3}}$.

The m. of m. of the planet's rotation is Cn,

and therefore $$h=C\left\{n+\mu^{\frac{2}{3}}\frac{Mm}{C}(M+m)^{-\frac{1}{3}}\Omega^{-\frac{1}{3}}\right\} \quad . \quad . \quad . \quad (1).$$

Again, the kinetic energy of orbital motion is

$$\tfrac{1}{2}M\left(\frac{Mr}{M+m}\right)^2\Omega^2+\tfrac{1}{2}m\left(\frac{Mr}{M+m}\right)^2\Omega^2=\tfrac{1}{2}\frac{Mm}{M+m}r^2\Omega^2=\tfrac{1}{2}\mu^{\frac{2}{3}}Mm(M+m)^{-\frac{1}{3}}\Omega^{\frac{2}{3}}.$$

The kinetic energy of the planet's rotation is $\frac{1}{2}Cn^2$.

The potential energy of the system is

$$-\mu\frac{Mm}{r}=-\mu^{\frac{2}{3}}Mm(M+m)^{-\frac{1}{3}}\Omega^{\frac{2}{3}}.$$

Adding the three energies together

$$2e=C\left\{n^2-\mu^{\frac{2}{3}}\frac{Mm}{C}(M+m)^{-\frac{1}{3}}\Omega^{\frac{2}{3}}\right\} \quad . \quad . \quad . \quad (2).$$

Now, suppose that by a proper choice of the unit of time, $\mu^{\frac{2}{3}}\frac{Mm}{C}(M+m)^{-\frac{1}{3}}$ is unity, and that by a proper choice of units of length or of mass C is unity,* and

Let $\qquad x=\Omega^{-\frac{1}{3}}, \qquad y=n, \qquad Y=2e.$

* Let $\nu=\frac{M}{m}$, then if g be the mean gravity at the surface of the planet, and if a be its mean radius,

$$\mu(M+m)=ga^2\frac{1+\nu}{\nu}$$

and $$\mu^{\frac{2}{3}}Mm(M+m)^{-\frac{1}{3}}=\left[ga^2\frac{1+\nu}{\nu}\right]^{\frac{2}{3}}\frac{M}{1+\nu}=Ma^2\div\left\{\left(\frac{a\nu}{g}\right)^2(1+\nu)\right\}^{\frac{1}{3}}$$

Then if the planet be homogeneous, and differ infinitesimally from a sphere, $C=\frac{2}{5}Ma^2$, and;

It may be well to notice that x is proportional to the square root of the satellite's distance from the planet.

Then the equations (1) and (2) become

$$h=y+x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3),$$

$$\mathrm{Y}=y^2-\frac{1}{x^2}=(h-x)^2-\frac{1}{x^2} \quad . \quad . \quad . \quad . \quad . \quad (4).$$

(3) is the equation of conservation of moment of momentum, or shortly, the equation of momentum; (4) is the equation of energy.

Now, consider a system started with given positive (or say clockwise) moment of momentum h; we have all sorts of ways in which it may be started. If the two rotations be of opposite kinds, it is clear that we may start the system with any amount of energy however great, but the true maxima and minima of energy compatible with the given moment of momentum are given by $\frac{d\mathrm{Y}}{dx}=0$,

or
$$x-h+\frac{1}{x^3}=0,$$

or
$$x^4-hx^3+1=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5).$$

We shall presently see that this biquadratic has either two real roots and two imaginary, or all imaginary roots.

This biquadratic may be derived from quite a different consideration, viz., by finding the condition under which the satellite may move round the planet, so that the planet shall always show the same face to the satellite, in fact, so that they move as parts of one rigid body.

The condition is simply that the satellite's orbital angular velocity

$$\mu^{\frac{2}{3}}\frac{Mm}{\mathrm{C}}(M+m)^{-\frac{1}{3}}=1\div\frac{2}{5}\left\{\left(\frac{a\nu}{g}\right)^2(1+\nu)\right\}^{\frac{1}{3}}=\frac{1}{s}, \text{ suppose}$$

in the case of the earth, considered as heterogeneous, the $\frac{2}{5}$ would be replaced by about $\frac{1}{3}$.

It is clear that $s^{\frac{3}{4}}$ is a time; and in the case of the earth and moon (with $\nu=82$),

$s^{\frac{3}{4}}=3$ hrs. $4\frac{1}{2}$ mins., if the earth be homogeneous, and

$s^{\frac{3}{4}}=2$ hrs. 41 mins. if the earth be heterogeneous.

For the units of length and mass we have only to choose them so that $\frac{2}{5}Ma^2$, or $\frac{1}{3}Ma^2$, may be unity.

With these units it will be found that for the present length of day $n=\cdot8056$ (homog.) or $\cdot7026$ (heterog.), and that

$$h=\cdot8056[1+4\cdot01]=4\cdot03 \text{ (homog.)},$$

$$\text{or } h=\cdot7026[1+4\cdot38]=3\cdot78 \text{ (heterog.)}$$

For the value 4·38 see Thomson and Tait's "Nat. Phil.," § 276, where tidal friction is considered.

$\Omega = n$ the planet's angular velocity round its axis; or since $n=y$ and $\Omega^{-\frac{1}{3}}=x$, therefore $y=\frac{1}{x^3}$.

By substituting this value of y in the equation of momentum (3), we get as before

$$x^4 - hx^3 + 1 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5).$$

In my paper on the "Precession of a Viscous Spheroid,"* I obtained the biquadratic equation from this last point of view only, and considered analytically and numerically its bearings on the history of the earth.

Sir William Thomson, having read the paper, told me that he thought that much light might be thrown on the general physical meaning of the equation, by a comparison of the equation of conservation of moment of momentum with the energy of the system for various configurations, and he suggested the appropriateness of geometrical illustration for the purpose of this comparison. The method which is worked out below is the result of the suggestions given me by him in conversation.

The simplicity with which complicated mechanical interactions may be thus traced out geometrically to their results appears truly remarkable.

At present we have only obtained one result, viz.: that if with given moment of momentum it is possible to set the satellite and planet moving as a rigid body, then it is possible to do so in two ways, and one of these ways requires a maximum amount of energy and the other a minimum; from which it is clear that one must be a rapid rotation with the satellite near the planet, and the other a slow one with the satellite remote from the planet.

Now, consider the three equations,

$$h = y + x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6),$$

$$Y = (h-x)^2 - \frac{1}{x^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7),$$

$$x^3 y = 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8).$$

(6) is the equation of momentum; (7), that of energy; and (8) we may call the equation of rigidity, since it indicates that the two bodies move as though parts of one rigid body.

Now, if we wish to illustrate these equations geometrically, we may take as abscissa x, which is the m. of m. of orbital motion; so that the axis of x may be called the axis of orbital momentum. Also, for equations (6) and (8) we may take as ordinate y, which is the m. of m. of the planet's rotation; so that the axis of y may be called the axis of rotational momentum. For (7) we may take as ordinate Y,

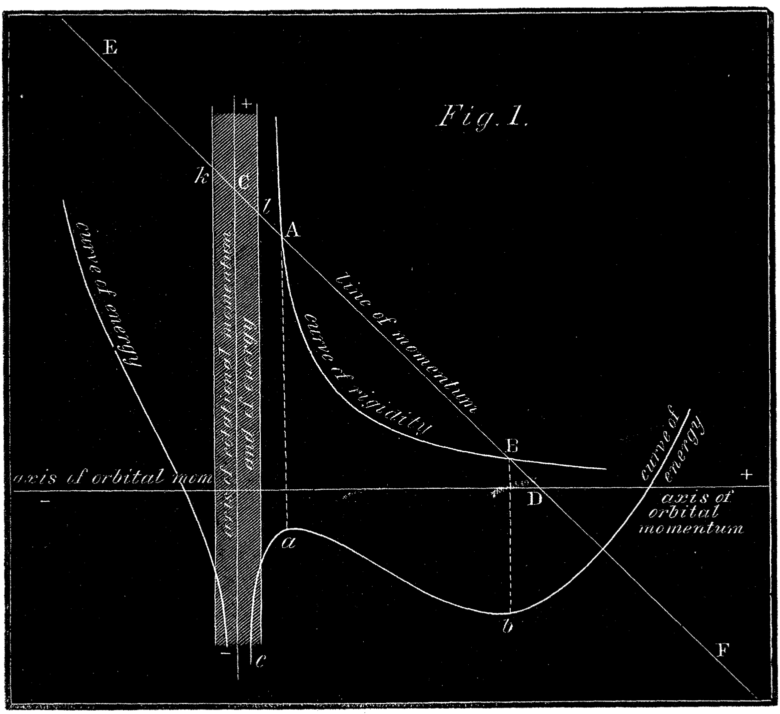
* Of which an abstract appears in "Proc. Roy. Soc.," No. 191, 1878.

which is twice the energy of the system; so that the axis of Y may be called the axis of energy. Then, as it will be convenient to exhibit all three curves in the same figure, with a parallel axis of x, we must have the axis of energy identical with that of rotational momentum.

It will not be necessary to consider the case where the resultant m. of m. h is negative, because this would only be equivalent to reversing all the rotations; thus h is to be taken as essentially positive.

Then the line of momentum, whose equation is (6), is a straight line at 45° to either axis, having positive intercepts on both axes.

The curve of rigidity, whose equation is (8), is clearly of the same nature as a rectangular hyperbola, but having a much more rapid rate of approach to the axis of orbital momentum than to that of rotational momentum.



The intersections (if any) of the curve of rigidity with the line of momentum have abscissæ which are the two roots of the biquadratic $x^4 - hx^3 + 1 = 0$. The biquadratic has, therefore, two real roots or all imaginary roots. Then, since $x = \Omega^{-\frac{1}{3}}$, it varies as $\sqrt{r}$, and, therefore, the intersection which is more remote from the origin, indicates a configuration where the satellite is remote from the planet; the other

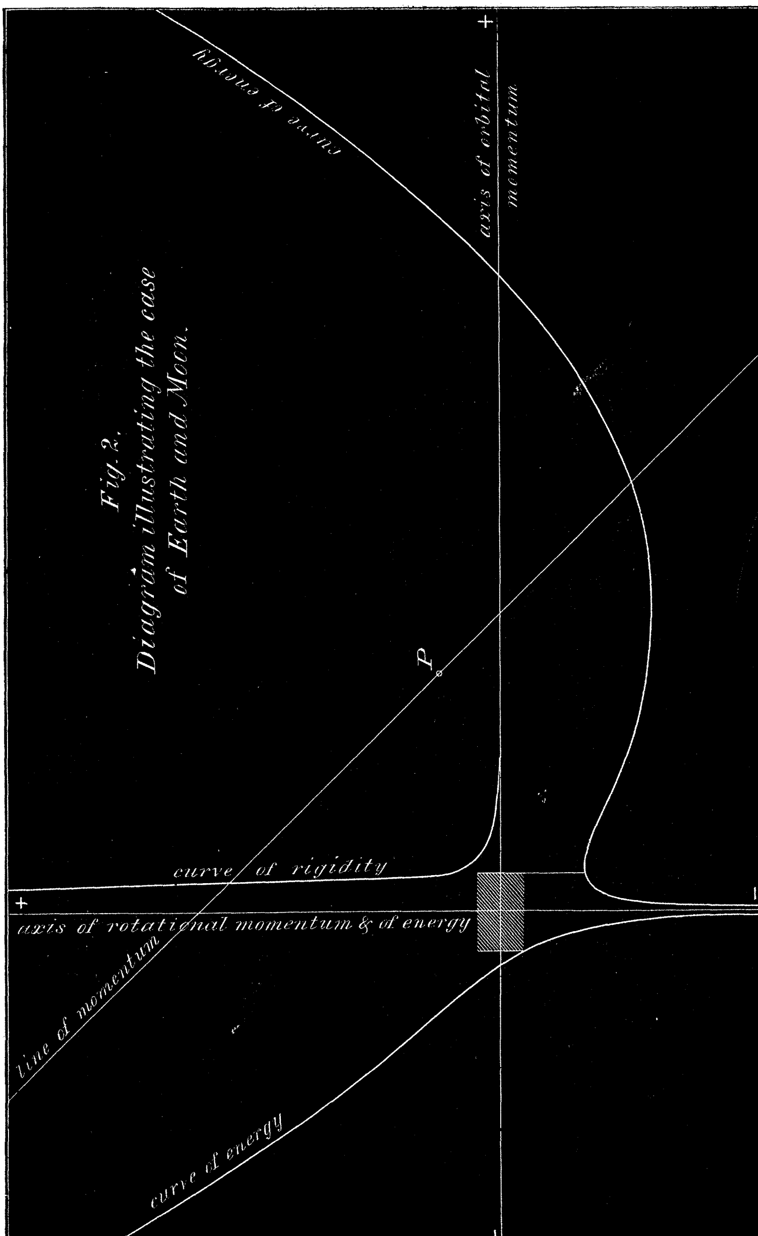


Fig. 2.
Diagram illustrating the case of Earth and Moon.

gives the configuration where the satellite is closer to the planet. We have already learnt that these two correspond respectively to minimum and maximum energy.

When x is very large, the equation to the curve of energy is $Y=(h-x)^2$, which is the equation to a parabola, with a vertical axis parallel to Y and distant h from the origin, so that the axis of the parabola passes through the intersection of the line of momentum with the axis of orbital momentum.

When x is very small the equation becomes $Y=-\frac{1}{x^2}$.

Hence, the axis of Y is asymptotic on both sides to the curve of energy.

Then, if the line of momentum intersects the curve of rigidity, the curve of energy has a maximum vertically underneath the point of intersection nearer the origin, and a minimum underneath the point more remote. But if there are no intersections, it has no maximum or minimum.

It is not easy to exhibit these curves well if they are drawn to scale, without making a figure larger than it would be convenient to print, and accordingly fig. 1 gives them as drawn with the free hand. As the zero of energy is quite arbitrary, the origin for the energy curve is displaced downwards, and this prevents the two curves from crossing one another in a confusing manner. The same remark applies also to figs. 2 and 3.

Fig. 1 is erroneous principally in that the curve of rigidity ought to approach its horizontal asymptote much more rapidly, so that it would be difficult in a drawing to scale to distinguish the points of intersection B and D.

Fig. 2 exhibits the same curves, but drawn to scale, and designed to be applicable to the case of the earth and moon, that is to say, when $h=4$ nearly.

Fig. 3 shows the curves when $h=1$, and when the line of momentum does not intersect the curve of rigidity; and here there is no maximum or minimum in the curve of energy.

These figures exhibit all the possible methods in which the bodies may move with given moment of momentum, and they differ in the fact that in figs. 1 and 2 the biquadratic (5) has real roots, but in the case of fig. 3 this is not so. Every point of the line of momentum gives by its abcissa and ordinate the square root of the satellite's distance and the rotation of the planet, and the ordinate of the energy curve gives the energy corresponding to each distance of the satellite.

Parts of these figures have no physical meaning, for it is impossible for the satellite to move round the planet at a distance which is less than the sum of the radii of the planet and satellite. Accordingly in fig. 1 a strip is marked off and shaded on each side of the vertical axis, within which the figure has no physical meaning.

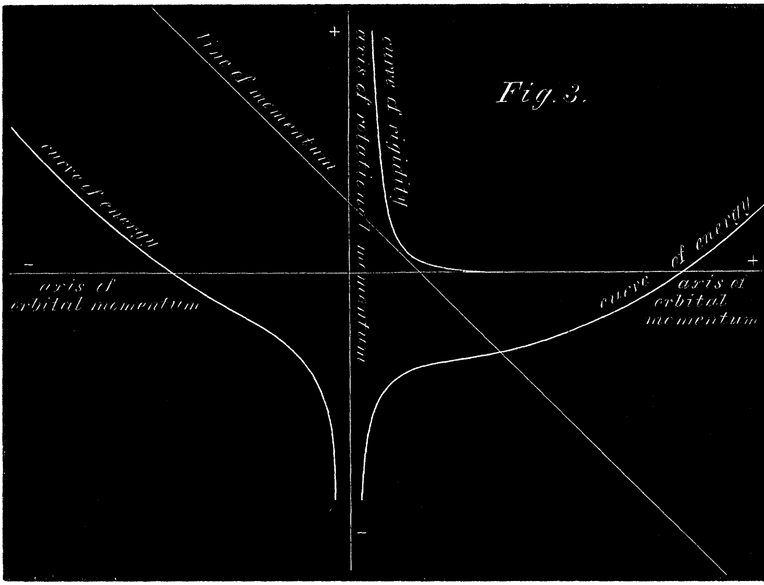


Fig. 3.

Since the moon's diameter is about 2,200 miles, and the earth's about 8,000, therefore the moon's distance cannot be less than 5,100 miles; and in fig. 2, which is intended to apply to the earth and moon and is drawn to scale, the base of the strip is only shaded, so as not to render the figure confused. The strip has been accidentally drawn a very little too broad.

The point P in fig. 2 indicates the present configuration of the earth and moon.

The curve of rigidity $x^3y=1$ is the same for all values of h, and by moving the line of momentum parallel to itself nearer or further from the origin, we may represent all possible moments of momentum of the whole system.

The smallest amount of m. of m. with which it is possible to set the system moving as a rigid body, is when the line of momentum touches the curve of rigidity. The condition for this is clearly that the equation $x^4-hx^3+1=0$ should have equal roots. If it has equal roots each root must be $\frac{3}{4}h$, and therefore

$$(\tfrac{3}{4}h)^4-h(\tfrac{3}{4}h)^3+1=0.$$

whence $h^4=\frac{4^4}{3^3}$ or $h=\frac{4}{3^{\frac{3}{4}}}=1{\cdot}75$.

The actual value of h for the moon and earth is about $3\frac{3}{4}$, and hence if the moon-earth system were started with less than $\frac{7}{15}$ of its actual moment of momentum, it would not be possible for the two bodies to move so that the earth should always show the same face to the moon.

Again if we travel along the line of momentum there must be some point for which yx^3 is a maximum, and since $yx^3=\frac{n}{\Omega}$ there must be some point for which the number of planetary rotations is greatest during one revolution of the satellite, or shortly there must be some configuration for which there is a maximum number of days in the month.

Now yx^3 is equal to $x^3(h-x)$, and this is a maximum when $x=\frac{3}{4}h$ and the maximum number of days in the month is $(\frac{3}{4}h)^3\ (h-\frac{3}{4}h)$ or $\frac{3^3}{4^4}h^4$; if h is equal to 4, as is nearly the case for the homogeneous earth and moon, this becomes 27.

Hence it follows that we now have very nearly the maximum number of days in the month. A more accurate investigation in my paper on the "Precession of a Viscous Spheroid," showed that taking account of solar tidal friction and of the obliquity to the ecliptic the maximum number of days is about 29, and that we have already passed through the phase of maximum.

We will now consider the physical meaning of the several parts of the figures.

It will be supposed that the resultant moment of momentum of the whole system corresponds to a clockwise rotation.

Now imagine two points with the same abscissa, one on the momentum line and the other on the energy curve, and suppose the one on the energy curve to guide that on the momentum line.

Then since we are supposing frictional tides to be raised on the planet, therefore the energy must degrade, and however the two points are set initially, the point on the energy curve must always slide down a slope carrying with it the other point.

Now looking at fig. 1 or 2, we see that there are four slopes in the energy curve, two running down to the planet, and two others which run down to the minimum. In fig. 3 on the other hand there are only two slopes, both of which run down to the planet.

In the first case there are four ways in which the system may degrade, according to the way it was started; in the second only two ways.

i. Then in fig. 1, for all points of the line of momentum from C through E to infinity, x is negative and y is positive; therefore this indicates an anti-clockwise revolution of the satellite, and a clockwise rotation of the planet, but the m. of m. planetary rotation is greater than that of the orbital motion. The corresponding part of the curve of energy slopes uniformly down, hence however the system be started, for this part of the line of momentum, the satellite must approach the planet, and will fall into it when its distance is given by the point k.

ii. For all points of the line of momentum from D through F to infinity, x is positive and y is negative; therefore the motion of the satellite is clockwise, and that of the planetary rotation anti-clockwise, but the m. of m. of the orbital motion is greater than that of the planetary rotation. The corresponding part of the energy curve slopes down to the minimum b. Hence the satellite must approach the planet until it reaches a certain distance where the two will move round as a rigid body. It will be noticed that as the system passes through the configuration corresponding to D, the planetary rotation is zero, and from D to B the rotation of the planet becomes clockwise.

If the total moment of momentum had been as shown in fig. 3, then the satellite would have fallen into the planet, because the energy curve would have no minimum.

From i and ii we learn that if the planet and satellite are set in motion with opposite rotations, the satellite will fall into the planet, if the moment of momentum of orbital motion be less than or equal to or only greater by a certain critical amount,* than the moment of momentum of planetary rotation, but if it be greater by more than a certain critical amount the satellite will approach the planet, the rotation of the planet will stop and reverse, and finally the system will come to equilibrium when the two bodies move round as a rigid body, with a long periodic time.

iii. We now come to the part of the figure between C and D. For the parts AC and BD of the line AB in fig. 1, the planetary rotation is slower than that of the satellite's revolution, or the month is shorter than day, as in one of the satellites of Mars. In fig. 3 these parts together embrace the whole. In all cases the satellite approaches the planet. In the case of fig. 3, the satellite must ultimately fall into the planet; in the case of figs. 1 and 2 the satellite will fall in if its distance from the planet is small, or move round along with the planet as a rigid body if its distance be large.

For the part of the line of momentum AB, the month is longer than the day, and this is the case of all known satellites except the nearer one of Mars. As this part of the line is non-existent in fig. 3, we see that the case of all existing satellites (except the Martian one) is comprised within this part of figs. 1 and 2. Now if a satellite be placed in the condition A, that is to say, moving rapidly round a planet, which always shows the same face to the satellite, the condition is clearly dynamically unstable, for the least disturbance will determine whether the system shall degrade down the slopes ac or ab, that is to say, whether it falls into or recedes from the planet. If

* With the units which are here used the excess must be more than $4 \div 3\frac{3}{4}$; see further back.

the equilibrium breaks down by the satellite receding, the recession will go on until the system has reached the state corresponding to B.

The point P, in fig. 2, shows approximately the present state of the earth and moon, viz., when $x=3{\cdot}2$, $y={\cdot}8$.

It is clear that, if the point l, which indicates that the satellite is just touching the planet, be identical with the point A, then the two bodies are in effect part of a single body in an unstable configuration. If, therefore, the moon was originally part of the earth, we should expect to find A and l identical. The figure 2, which is drawn to represent the earth and moon, shows that there is so close an approach between the edge of the shaded band and the intersection of the line of momentum and curve of rigidity, that it would be scarcely possible to distinguish them on the figure. Hence, there seems a considerable probability that the two bodies once formed parts of a single one, which broke up in consequence of some kind of instability. This view is confirmed by the more detailed consideration of the case in the paper on the "Precession of a Viscous Spheroid," of which only an abstract has as yet been printed.

Hitherto the satellite has been treated as an attractive particle, but the graphical method may be extended to the case where both the satellite and planet are spheroids rotating about axes perpendicular to the plane of the orbit.

Suppose, then, that k is the ratio of the moment of inertia of the satellite to that of the planet, and that z is equal to the angular velocity of the satellite round its axis, then kz is the moment of momentum of the satellite's rotation, and we have

$h=x+y+kz$ for the equation to the plane of momentum,

$2e=y^2+kz^2-\dfrac{1}{x^2}$ for the equation of energy,

and $x^3y=1$, $x^3z=1$ for the equation to the line of rigidity.

The most convenient form in which to put the equation to the surface of energy is

$$E=y^2+kz^2-\frac{1}{(h-y-kz)^2},$$

where E, y, z are the three ordinates.

The best way of understanding the surface is to draw the contour-lines of energy parallel to the plane of yz, as shown in fig. 4.

The case which I have considered may be called a double-star system, where the planet and satellite are equal and $k=1$. Any other case may be easily conceived by a stretching or contraction of the surface parallel to z.

It will be found that, if the whole moment of momentum h has less than a certain critical value (found by the consideration that

$x^4 - hx^3 + 2 = 0$ has equal roots), then the surface may be conceived as an infinitely narrow and deep ravine, opening out at one part of its course into rounded valleys on each side of the ravine. In this case the contours would resemble those of fig. 4, supposing the round closed curves to be absent. The course of the ravine is at 45° to the axes of y and z, and the origin is situated in one of the valleys, which is less steep than the valley facing it on the opposite side of the ravine. The form of a section perpendicular to the ravine is such as the curve of energy in fig. 3, so that everywhere there is a slope towards the ravine.

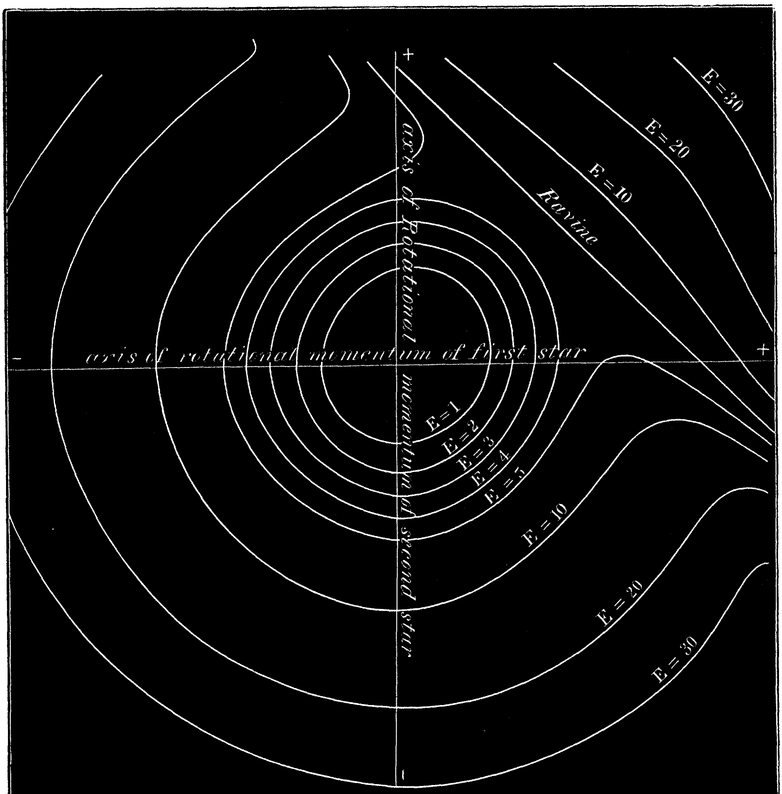


Fig. 4
Contour lines of energy surface for two equal stars, revolving about one another.

Every point on the surface corresponds to one configuration of the system, and, if the system be guided by a point on the energy surface, that point must always slide down hill. It does not, however, necessarily follow that it will always slide down the steepest path. The fall of the guiding point into the ravine indicates the falling together of the two stars.

Thus, if the two bodies be started with less than a certain moment of momentum, they must ultimately fall together.

Next, suppose the whole m. of m. of the system to be greater than the critical value. Now, the less steep of the two valleys of the former case (viz., the one in which the origin lies) has become more like a semicircular amphitheatre of hills, with a nearly circular lake at the bottom; and the valley facing the amphitheatre has become merely a falling back of the cliffs which bound the ravine. The energy curve in fig. 2 would show a section perpendicular to the ravine through the middle of the lake.

The origin is nearly in the centre of the lake, but slightly more remote from the ravine than the centre.

In this figure h was taken as 4, and k as unity, so that it represents a system of equal double stars. The numbers written on each contour give the value of E corresponding to that contour.

Now, the guiding point of the system, if on the same side of the ravine as the origin, may either slide down into the lake or into the ravine. If it falls into the ravine, the two stars fall together, and if to the bottom of the lake, the whole system moves round slowly, like a rigid body.

If the point be on the lip of the lake, with the ravine on one side and the lake on the other, this corresponds to the motion of the two bodies rapidly round one another, moving as a rigid body; and this state is clearly dynamically unstable.

If the point be on the other side of the ravine, it must fall into it, and the two stars fall together.

It has been remarked that the guiding point does not necessarily slide down the steepest gradient, and of such a mode of descent illustrations will be given hereafter.

Hence it is possible that, if the guiding point be started somewhere on the amphitheatre of hills, it may slide down until it comes to the lip of the lake. As far as one can see, however, such a descent would require a peculiar relationship of the viscosities of the two stars, probably varying from time to time. It is therefore possible, though improbable, that the unstable condition where the two bodies move rapidly round one another, always showing the same faces to one another, may be a degradation of a previous condition. If this state corresponds with a distance between the stars less than the sum of the radii of their masses, it clearly cannot be the result of such a degradation.

If, therefore, we can trace back a planet and satellite to this state, we have most probably found the state where the satellite first had a separate existence.

The conditions of stability of a rotating mass of fluid are very obscure, but it seems probable that, if the stability broke down and the mass gradually separated into two parts, then the condition immediately after separation might be something like the unstable configuration described above.

In conclusion, I will add a few words to show that the guiding point on an energy surface need not necessarily move down the steepest path, but may even depart from the bottom of a furrow or move along a ridge. Of this two cases will be given.

The satellite will now be again supposed to be merely an attractive particle.

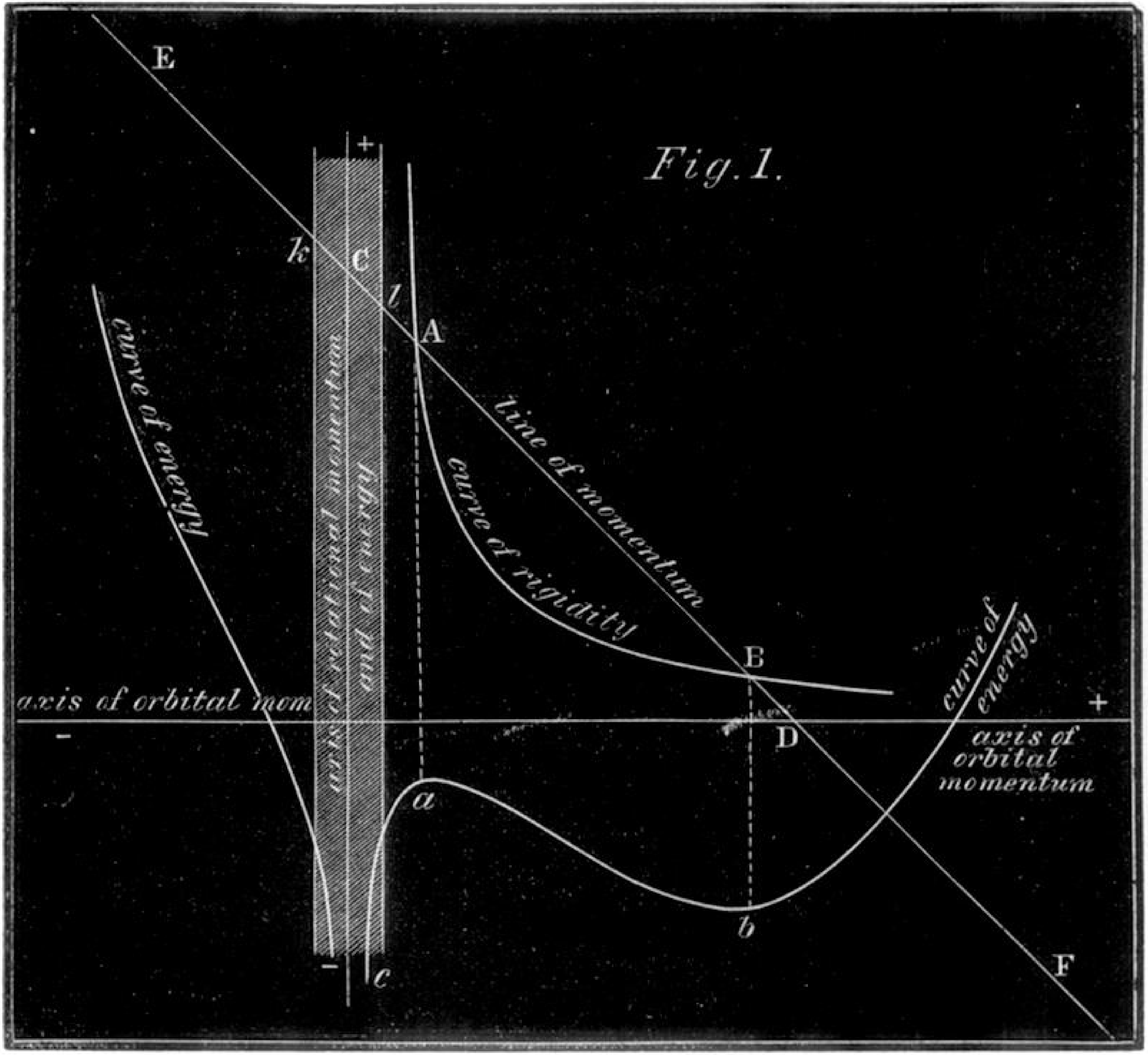
First, with given moment of momentum, the energy is greater when the axis of the planet is oblique to the orbit. Hence, if we draw an energy surface in which one of the co-ordinate axes corresponds to obliquity, then there must be a furrow in the surface corresponding to zero obliquity. To conclude that the obliquity of the ecliptic must diminish in consequence of tidal friction would be erroneous. In fact, it will appear, in my paper on the "Precession of a Viscous Spheroid," that for a planet of small viscosity the position of zero obliquity is dynamically unstable, if the period of the satellite is greater than twice that of the planet's rotation. Thus the guiding point, though always descending on the energy surface, will depart from the bottom of the furrow.

Secondly. For given moment of momentum the energy is less if the orbit be eccentric, and an energy surface may be constructed in which zero eccentricity corresponds to a ridge. Now, I believe that I shall be able to show, in a future paper, that for small viscosity of the planet the circular orbit is dynamically stable if eighteen periods of the satellite be less than eleven periods of the planet's rotation. This will afford a case of the guiding point sliding down a ridge; when, however, the critical point is passed, the guiding point will depart from the ridge and the orbit become eccentric.

IX. "Researches in Chemical Equivalence. Part III.* Nickelous and Cobaltous Sulphates." By Edmund J. Mills, D.Sc., F.R.S., and J. J. Smith. Received June 2, 1879.

Although the chemistry of nickel and cobalt is interesting from many points of view, it is more especially attractive from the probable isomerism of these metals. Their combining proportions, in fact,

* For Part II, see "Proceedings," vol. xxviii, p. 270.


E
k
C
l
A
+
curve of energy
line of momentum
curve of rigidity
B
axis of orbital mom
-
D
+
curve of energy
axis of orbital momentum
a
b
c
-
F
Fig. 1.

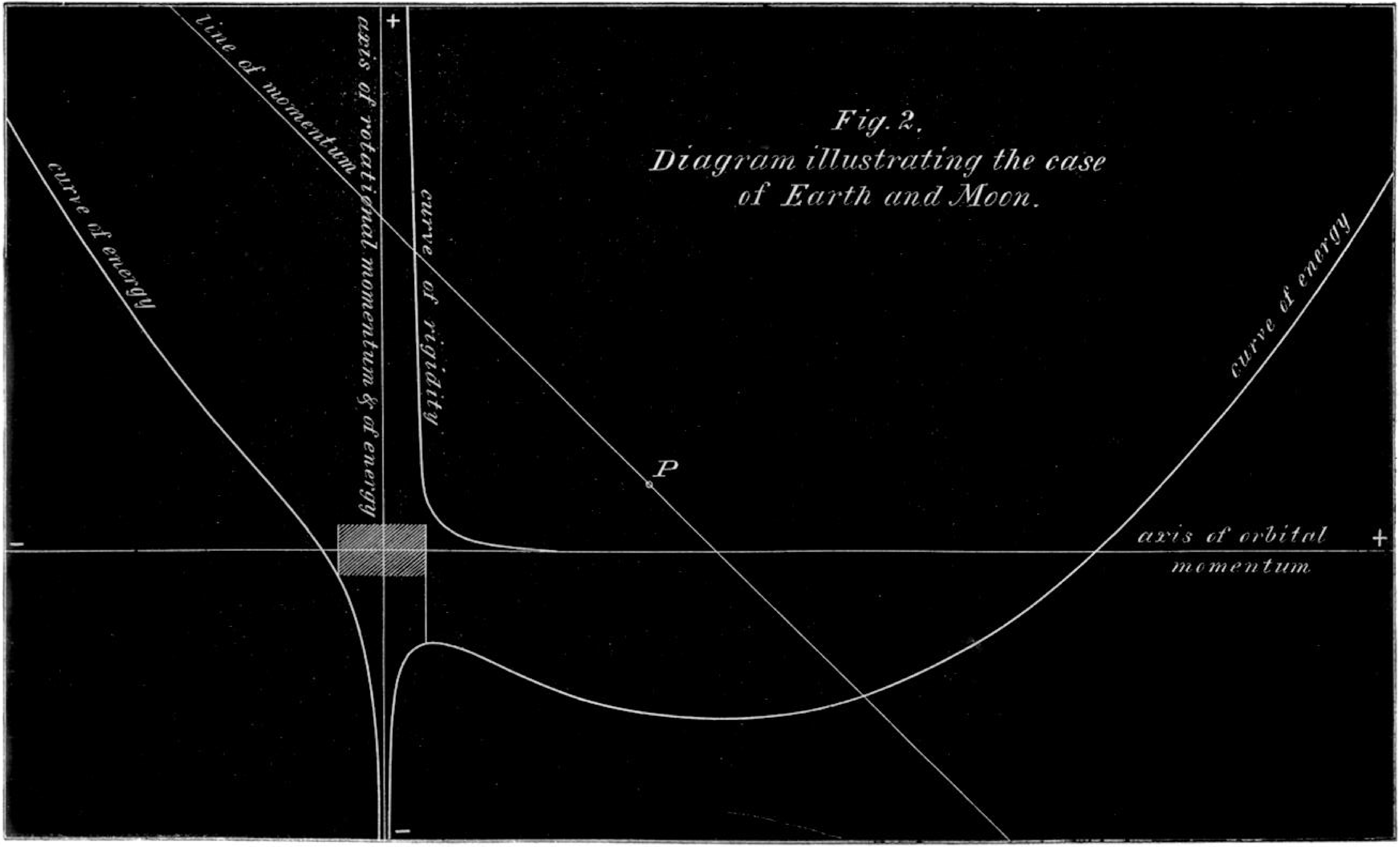


Fig. 2.
Diagram illustrating the case
of Earth and Moon.

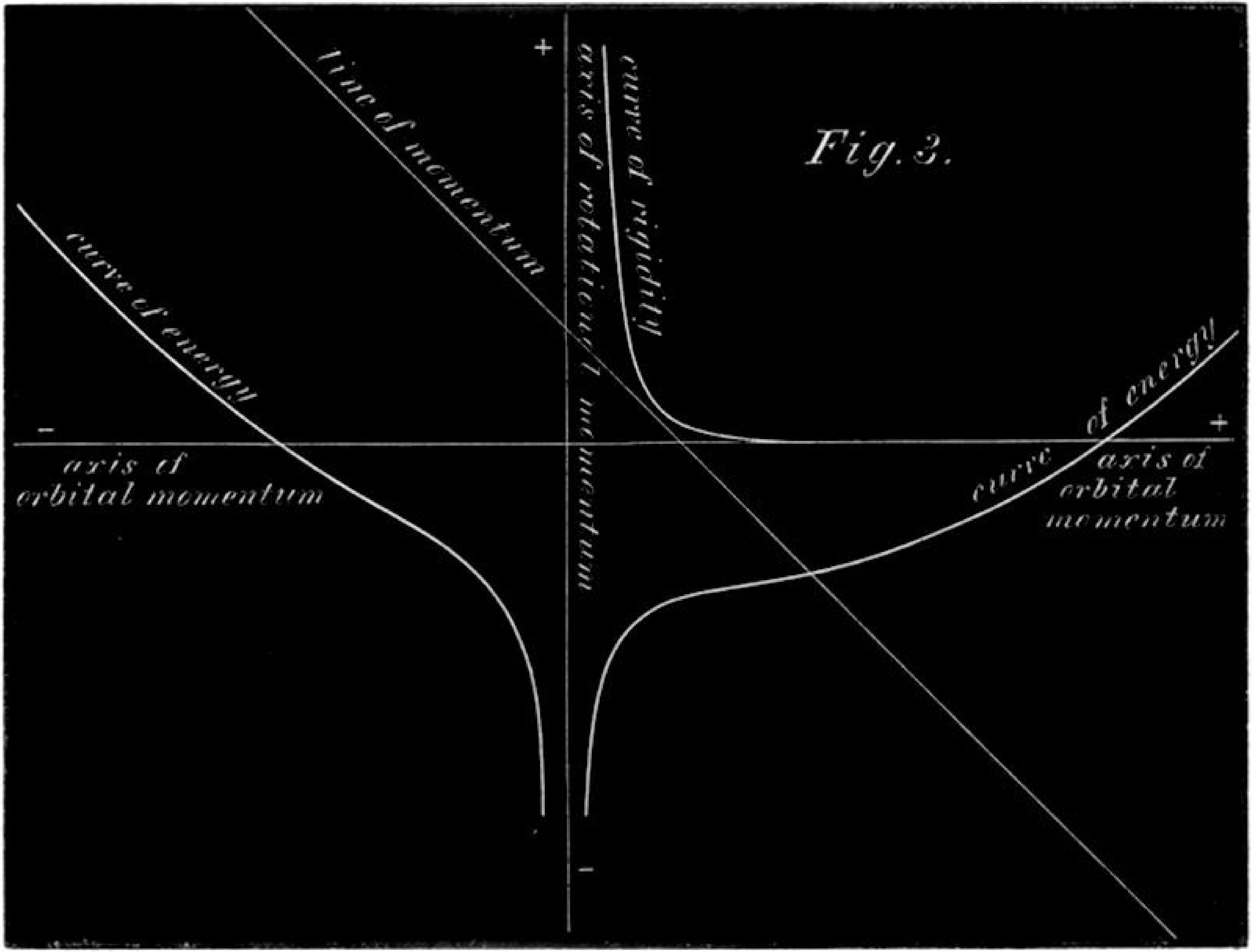


Fig. 3.
line of momentum
axis of rotational momentum
curve of rigidity
curve of energy
curve of energy
axis of orbital momentum
axis of orbital momentum
+
+
-
-

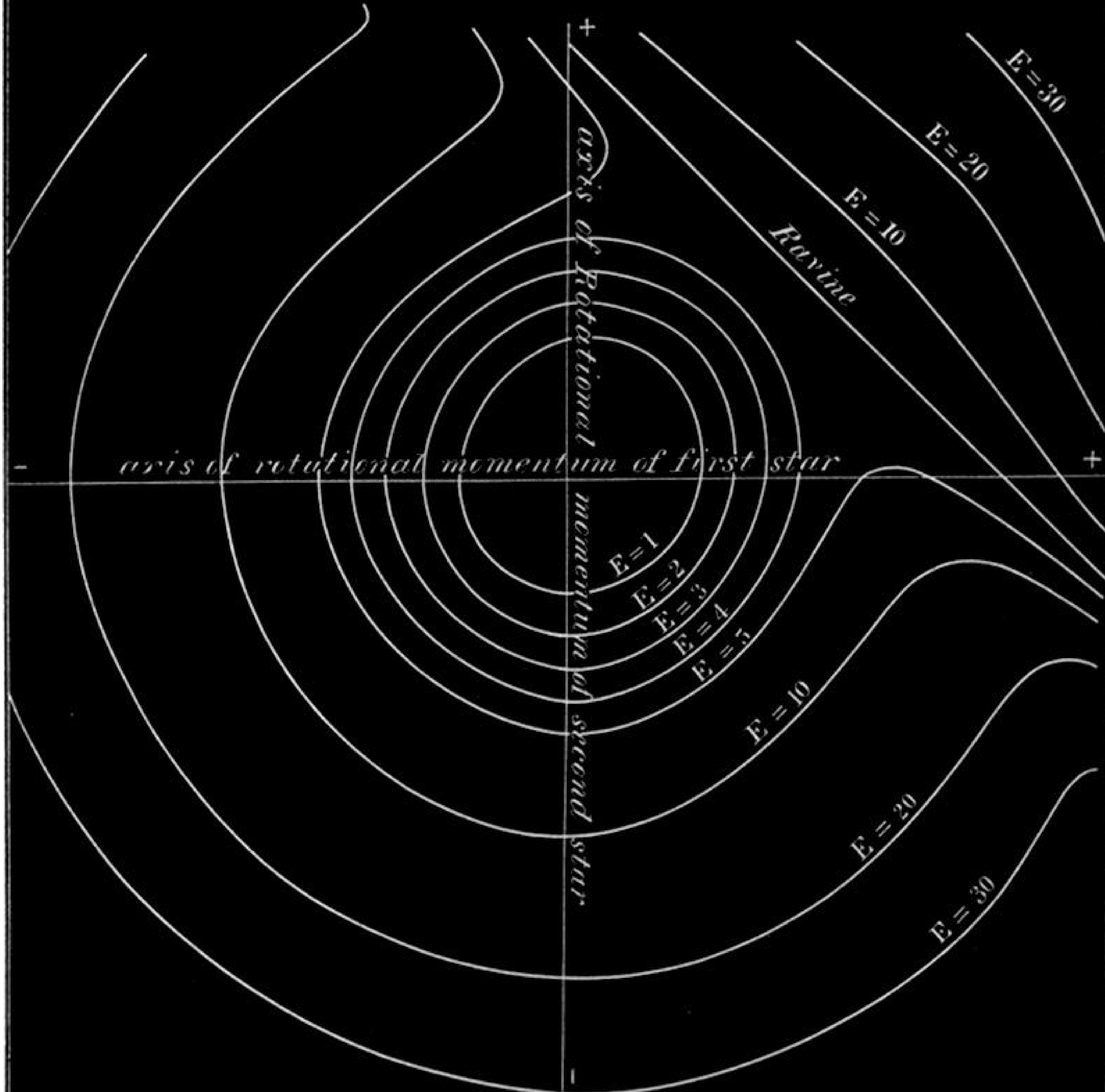


Fig. 4
Contour lines of energy surface
for two equal stars,
revolving about one another.