[viz., the equation $(1-u^8)(1-v^8)-(1-uv)^8=0$ is $u^8+v^8-\Theta=0$], and the modular equation as obtained by the elimination from the two quadric equations in fact presents itself in the form

$$(u^4-v^4+2\theta-2\theta^3)^2(u^4-v^4-2\theta+2\theta^3)^2(u^8+v^8-\Theta)=0.$$

March 14, 1878.

Sir JOSEPH HOOKER, K.C.S.I., President, in the Chair.

The Presents received were laid on the table and thanks ordered for them.

The following Papers were read:—

I. "On Professor Haughton's Estimate of Geological Time." By GEORGE H. DARWIN, M.A., Fellow of Trinity College, Cambridge. Communicated by J. W. L. GLAISHER, M.A., F.R.S. Received February 19, 1878.

In a paper recently read before the Royal Society,* Professor Haughton has endeavoured by an ingenious line of argument to give an estimate of the time which may have elapsed in the geological history of the earth. The results attained by him are, if generally accepted, of the very greatest interest to geologists, and on that account his method merits a rigorous examination. The object, therefore, of the present note is to criticise the applicability of his results to the case of the earth; and I conceive that my principal criticism is either incorrect, and will meet its just fate of refutation, or else is destructive of the estimate of geological time.

Professor Haughton's argument may be summarised as follows:—The impulsive elevation of a continent would produce a sudden displacement of the earth's principal axis of greatest moment of inertia. Immediately after the earthquake, the axis of rotation being no longer coincident with the principal axis, will, according to dynamical principles, begin describing a cone round the principal axis, and the complete circle of the cone will be described in about 306 days. Now, the ocean not being rigidly connected with the nucleus, a 306-day tide will be established, which by its friction with the ocean bed will tend

* "Notes on Physical Geology. No. III. On a New Method of finding Limits to the Duration of certain Geological Periods." "Proc. Roy. Soc.," vol. xxvi, pp. 534—546 (December 20, 1877).

to diminish the angle of the cone described by the instantaneous axis round the principal axis: in other words, the "wabble" set up by the earthquake will gradually die away.

Then by means of Adams and Delaunay's estimate of the alteration of the length of day, which is attributed to tidal friction, Professor Haughton obtains a numerical value for the frictional effect of the residual tidal current. He then applies this to the 306-day tide, and deduces the time required to reduce a "wabble" of given magnitude to any given extent.

He is of opinion that if, at the present time, the instantaneous axis of rotation of the earth were describing a circle of more than 10 feet in diameter at the earth's surface, then the phenomenon could not escape detection by modern astronomical instruments. From the absence of any such inequality he concludes, after numerical calculation, "if Asia and Europe were manufactured *per saltum*, causing a sudden displacement of the axis of figure through 69 miles, that this event cannot have happened at an epoch less than 641,000 years before the present time, and that this event may have occurred at an epoch much more remote."

He then passes on to consider the case where the elevation takes place by a number of smaller impulses instead of by one large one. He treats first the case of "69 geological convulsions, each of which displaced the axis of figure through one mile," and where "the radius of the wabble" is "reduced from one mile to 5 feet in the interval between each two successive convulsions;" and, secondly, the case where "the increase of this radius is exactly destroyed by friction during each wabble, so that the radius of 5 feet continues constant."

In the first case he finds that the total time occupied by the manufacture of Europe and Asia is $27\frac{1}{2}$ millions of years, and also that "no geological change, altering the position of the axis of figure through one mile, can have taken place within the past 400,000 years." And in the second case, he finds that the same elevation would occupy 4,170 millions of years. A little lower he adds: "It is extremely improbable that the continent of Asia and Europe was formed *per saltum*, and therefore our minor limit of time is probably far short of the reality."

It appears from these passages that Professor Haughton is of opinion that a succession of smaller impulses at short intervals will necessarily increase the radius of the "wabble;" but it is not very clear to me whether he means that the radius of the "wabble" would be the same by whatever series of impulses the principal axis was moved from one position to another. Now, I conceive that it is by no means necessary that a second impulse succeeding a first should augment the radius of the "wabble;" it might, indeed, annihilate it. I admit that by properly timed impulses the radius of the "wabble" might be made as

great as if the whole change took place by a single convulsion. But where the impulses take place at hazard there will be a certain average effect on the radius of the "wabble," which, as far as I can see, Professor Haughton makes no attempt to determine. It seems, therefore, an unjustifiable assumption that sufficient time must elapse between the successive impulses to reduce the radius of the "wabble" to 5 feet, for if the impulses took place more frequently they might tend to some extent to counteract one another. If this assumption is unjustifiable, then Professor Haughton's estimate of time falls with it.

In my paper on the "Influence of Geological Changes on the Earth's Axis of Rotation,"* I have considered the effects of a slow continuous distortion of the earth. The results there attained would, of course, have been identical, had I considered the effects of a series of infinitely small and infinitely frequent earthquakes. I presume Professor Haughton will agree with me in thinking this supposition more consonant with geological science than the larger earthquakes which he postulates.

I will now show, from the results of my paper, that *without calling in any effects whatever of tidal friction*, Asia and Europe might have been gradually upheaved in 19,200 years, without leaving any "wabble" sufficiently large to be detected astronomically, and, moreover, that at no time during the elevation could the "wabble" have been detected had astronomers been in existence to make observations; and further, that under certain not improbable suppositions, this estimate of time may be largely reduced. Let a be the angular velocity of the principal axis relatively to the solid earth, arising from the continuous elevation of the continent; n the earth's angular velocity of rotation; C, A the greatest and least principal moments of inertia of the earth; and $\mu = \frac{C-A}{A} n$.

Then, in section 2 of my paper, I show that the extremity of the instantaneous axis describes a circle at the earth's surface in 306 days, and that this circle passes through the extremity of the principal axis, and touches the meridian along which the principal axis is travelling with velocity a in consequence of the postulated geological change. Strictly speaking, the curve described by the instantaneous axis, is a trochoid, because the circle travels in the earth along with the principal axis; but the motion of the circle is so slow compared with that of the instantaneous axis along its arc, that it is more convenient to say that the instantaneous axis describes a circle which slowly changes its position. It must be noticed that this circle is unlike the "wabble" considered by Dr. Haughton, inasmuch as the extremity of the principal axis lies on its arc instead of being at its

* "Phil. Trans.," vol. clxvii, pt. I, p. 271.

centre. It is also shown in the same section that the diameter of the circle is equal to $\frac{2a}{\mu}$.

I will now suppose that the geological changes begin suddenly from rest, and proceed at such a rate that the variations in the position of the principal axis are imperceptible to astronomical observation. I will suppose, therefore, that the extremity of the instantaneous axis is never more than 5 feet distant from the extremity of the principal axis. Now, 5 feet at the earth's surface, subtends very nearly ·05″ at the earth's centre, and, therefore, to find a on this supposition, $\frac{2a}{\mu}$ must be put equal to ·05″.

μ is an angular velocity of 360° in 306 days, and if we wish to express a in seconds of arc per annum, μ must be expressed in those units, and ·05″ must be expressed in circular measure. Thus

$$a=\frac{1}{2}\times\cdot05\times\frac{\pi}{648000}\times360\times60\times60\times\frac{365\cdot25}{306}$$
$$=\frac{18\cdot263}{306}\pi=\frac{3}{16}\text{ very nearly.}$$

Therefore, a is an angular velocity of 1° (or 69 miles) in 19,200 years.

But, according to Professor Haughton, 69 miles is the displacement of the earth's principal axis, due to the elevation of Europe and Asia; hence, at this rate of elevation, Europe and Asia would have been heaved up in 19,200 years.

Now, if the elevation be supposed to stop suddenly, then the instantaneous axis cannot, at the time of the stoppage, be more than 5 feet distant from the axis of figure, and it may even be coincident with it. Therefore the stoppage cannot set up a "wabble" of more than 10 feet in diameter, and it may set up none at all. But even this maximum "wabble" of 10 feet, would, according to Professor Haughton, be imperceptible, and *à fortiori* the circle of 5 feet in diameter, described in the course of the elevation, would be imperceptible.

On any of the following suppositions, the elevation might be much more rapid, without increasing the residual "wabble":—

(1.) The stoppage of the elevation to take place at a time when the instantaneous axis is separated from the principal axis by a small angle.

(2.) The elevation partly counterbalanced by simultaneous elevations in other parts of the world, so that the upheaval of Europe and Asia would not displace the pole of figure by so much as 69 miles.

(3.) The elevation partly or altogether produced by the intumescence of the strata immediately underlying those continents. (See Part VI of my paper above referred to.)

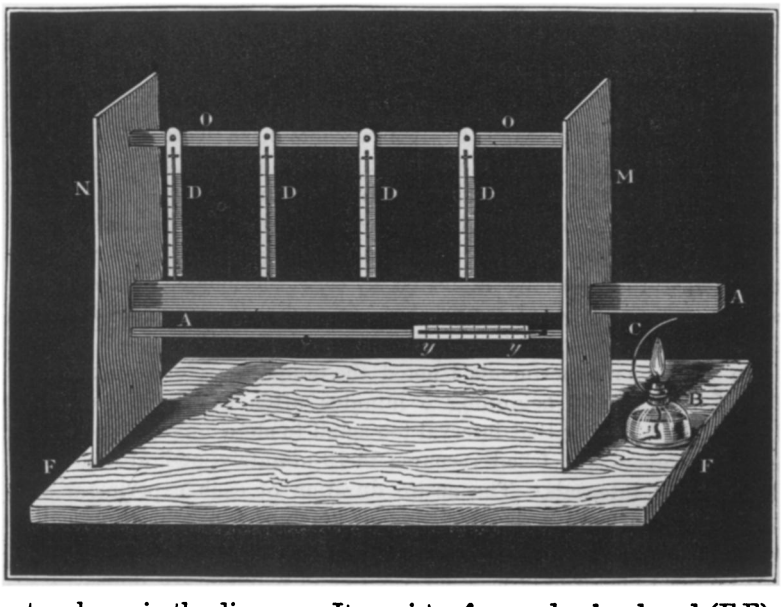
(4.) The elevation not uniform but more rapid in the earlier

portion of the time, so that the magnitude of the "wabble" would be reduced by the friction of the 306-day tide; for we are by no means compelled to believe that that inequality of motion must always have remained as small as it is at present.

It appears to me, from these considerations, that the continents of Europe and Asia might have been elevated in very much less than 20,000 years, and yet leave no record of the fact in the present motion of the earth. Therefore, if my solution of the problem is correct, it is certain that Professor Haughton's method can give us no clue to the times which have elapsed in the geological history of the earth.

II. "Some Experiments on Conductive Properties of Ice, made in Discovery Bay, 1875–6." By Staff Surgeon R. W. COPPINGER, M.D. Communicated by Professor TYNDALL, F.R.S. Received February 21, 1878.

In attempting a series of experiments for determining the rate of conduction of heat through sea and fresh-water ice, I have endeavoured as closely as possible to follow the suggestions made by Professor Tyndall in page 34 of the "Scientific Instructions for the Arctic Expedition." With this view, I have constructed the simple appa-



ratus shown in the diagram. It consists of a wooden baseboard (F F), 29 inches in length by 9 in breadth, on which are fixed two upright